Physics of Fashion Fluctuations

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Abstract: We consider a market where many agents trade many different types of products with each other. We model development of collective modes in this market, and quantify these by fluctuations that scale with time with a Hurst exponent of about 0.7. We demonstrate that individual products in the model occationally become globally accepted means of exchange, and simultaneously become very actively traded. Thus collective features similar to money spontaneously emerge, without any a priori reason.

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One of the most important interactions among humans is the exchange of goods and services. Trade has a number of benefits. Besides strengthening social contacts, it directly allows individuals to specialize while still benefitting from a large diversity of products. Another important benefit is risk minimization. Even in primitive markets it is beneficial for agents to have access to a diversity of goods, thereby minimizing their dependence on any particular one.

In modern markets risk minimization is similarly obtained by diversification of stock portfolios. However, diversification may be orthogonal to maximizing short term profit, and the frustration between these two goals strongly influences the behaviour of investors.

In the present paper we discuss a recently proposed minimal model for a market where agents attempt both to obtain the goods they need, by maintaining a stock of different products, and, at the same time, obtain what other agents want [1]. In other terms, they try to fulfill both their "need" and their "greed". The model was inspired by Yasutomi's work [2] on possible emergence of certain goods as "money" in the sense that they become fashionable, and therefore desirable, in a market where only barter trade takes place [3].

The minimal model for the market we propose consists of N_{ag} agents and N_{pr} different products. Initially we give N_{unit} units of the products to each agent. The number N_{unit} is fixed, but the products are chosen at random, so the individuals are not in exactly the same situation. At each timestep we select two agents at random. These will attepmt a trade between themselves. The trade starts by comparing the list of goods that each agent lacks and therefore would like to get from the other agent in exchange for goods it has in stock.

We first consider the simple "need" based exchange procedure: when each of the agents has products that the other needs, then one of these products, chosen at random, is exchanged.

In case such a "need" based exchange is not possible we consider the "greed" exchange procedure: one or both of the agents accept goods which they do not lack, but consider useful for future exchanges.

In order to determine the usefulness of a product, each agent i keeps a record of the

last requests for goods he received in encounters with other agents. This memory is finite, having a length of $N_{mem}(i)$ positions, each of which registers a product that was requested. Different agents may in principle have different memory length, although we typically will set all to be equal, i.e., $N_{mem}(i) = N_{mem}$ for all i. As the memory gets filled, the record of old transactions is lost.

The demand for a product is measured by each agent through the number of times that product appears on its memory list. Agents will then accept products they already have in stock with a probability based on its memory record. The chance that agent i accepts a good j is taken to be proportional to the number of times T_{ij} that good j appears on the memory list of agent i:

$$p_{ij} \; = \; \frac{T_{ij}}{N_{mem}(i)} \; , \tag{1}$$

where we have used the fact that $\sum_j T_{ij} = N_{mem}(i)$. These two exchange mechanisms define our model.

In a previous paper [1] we have demonstrated that after a small number of encounters per agent, the initial needs of all agents become locally minimized and most trades are based on the “greed” procedure.

As mentioned above, we typically we assign the same memory to all agents. However, as an initial lesson, we would like to demonstrate that memory helps agents in their trading. To do this, we consider a system where $N_{ag} = 200$ agents trade $N_{pr} = 200$ different types of products, and where each agent is given $N_{unit} = 400$ units of products. To illustrate the effect of memory, we have performed two different calculations, in which we arbitrarily assigned to each agent i either a memory $N_{mem}(i) = i$ or $N_{mem}(i) = 5i$.

Fig. 1 shows the time averaged trading probability of agents as function of their memory. We notice that, in general, trading activity increases with memory size. Thus, a long memory provides the agent with a better perception of the overall needs in the market, which leads them to value products others agents also have in high demand. One should note, however, that agents with very short memories also do well, which probably means that an agent that

trades practically at random may do better than agents which know less than the typical trader. We should stress that what agents are optimizing here is their trading activity, and not their individual wealth. Therefore this lesson may apply to agents that work for a commision on transactions between real traders.

For the remaining of this paper we consider that all agents have the same memory size, $N_{mem}(i) = N_{mem}$, and use the model to examine how different agents allocate their stocks, and how particular products change from being fashions to being forgotten.

We now study some specific properties of the model. After the initial reshuffling of goods, during which, as we have mentioned, mostly basic needs are fulfilled, each product will begin a slow development towards an absorbing state, i.e., a state which will develop no further when reached. Each product reaches this absorbing state when all agents have the product, and, at the same time, none of the agents remembers that any other agents needs the product, i.e., the product has disappeared from the memory of all agents. In this case all activity on that product disappears.

In Fig. 2 we show the number of active products as function of number of trades per agent (which is a measure of elapsed time) for a system with size parameters $N_{ag} = 50$, $N_{pr} = 50$, $N_{mem} = 100$ and $N_{unit} = 100$. The upper curves in fig. 2 indicate that the decay towards the absorbing state is avoided through the inclusion of production/consumption processes. Production/consumption is here defined as a probability p that, at a given time, an agent consumes a random product in his stock, and produces another he either needs or considers valuable. The rate of production p in the two cases shown are one per 100 trades ($p = 0.01$) and one per 1000 trades ($p = 0.001$) in the system.

When noticing the tendency of a few products to dominate the market, discussed in Ref. [1], one would similarly observe that production limits this tendency. This is easily understood, because, if products are easily produced, they do not become needed and therefore valuable. Furthermore, we have checked that, if different products are produced with rates which are different a priori, it is the slowly produced products that become valuable.

The freezing of products into the absorbing state, implies that demand tends to concen-

trate on just a few products, i.e., it emerges a subset of goods that are globally accepted for trade. During the slow transient towards the frozen state the total demand for a product j

$$D(j) \;=\; \sum_i T_{ij} \;, \tag{2}$$

fluctuates as a function of the number of trades t as

$$\Delta D(j) \propto (\Delta t)^H \;, \tag{3}$$

i.e., as a fractional Brownian walk with Hurst exponent H [4,5]. We have found that, in the case of no production/consumption processes, for a large range of system sizes, $H = 0.7 \pm 0.05$ [1].

In the case of finite production we have also found that fluctuations in demand also exhibit fractional Brownian walk properties in the statistically stationary state. This is illustrated in Fig. 3 a and b where we show, for a given product, the time series of its demand and its fluctuation statistics, respectively. The calculations were done for the same system parameters as in Fig. 2, and with production/consumption rate $p = 0.001$.

We have thus seen that if we quantify the value of products through their demand, then value becomes concentrated on a few products, and exhibits persistent fluctuations (*i.e.* fluctuations with Hurst exponent $H > 0.5$). Such basic features are commonly found in goods that are employed as means of exchange, that is, serve as money. In order to quantify this common view of the what is valuable, we define the monetary value of a good j as the number of agents which consider that good as the most wanted in the system,

$$M(j) = \sum_i \prod_{j' \neq j} S(T_{ij} - T_{ij'}) \;, \tag{4}$$

where $S(x)$ is the step function, $S(x \geq 0) = 1, S(x < 0) = 0$.

In Fig. 4 we examine the monetary value as function of demand for the products in the system considered in Fig. 3. We observe a clearly nonlinear relationship between monetary value and demand, which reflects the extremal nature of M. We also notice that, when demand exceeds a certain critical value, the product becomes the most globally accepted

means of exchange, or, in more popular terms it becomes money. As a consequence of this nonlinear relationship, the short time fluctuations of monetary value of goods exhibit pronounced fat tails, which gradually become Gaussian when considered over longer time intervals [1].

The increased demand for a product is closely linked to an increased heterogeneity of the distribution of products between agents. As a product becomes slightly more needed than the rest, agents start demanding it, and accepting it even without need because they sense other agents needing it. As a result of the heavy trading in that good, and the resulting fluctuations in its distribution throughout the system, the good may be accumulated by a few agents, which further amplifies its need. In the lower curves of Fig. 4 we show that demand for a product is proportional to the number of agents that lack it. The two lower curves show respectively the instantaneous number of agents without the product and the number of agents that persistently over a time interval $\Delta t = 2$ do not have the product. This shows that although the most valuable product is unevenly distributed, the ownership will fluctuate on a fast timescale.

Finally note that the same fluctuations that make a good the most traded may also distribute it evenly among agents. If the distribution lasts long enough for other products to replace it in the memory of the agents in the system, another product will become the most fashionable. Thus, the concept of money, and its stability, may be directly linked to the fact that economic agents have memory.

For random appearance of monetary systems, consult the case studies of Yasutomi [2]. For quantifications of persistent fluctuations of value in stock and monetary markets, see e.g., Ref. [6]. For discussions of other models which exhibit anomalous scaling, see also [1,7,8] whereas a detailed discussion of scaling phenomena in economy may be found in the excellent review of Farmer [9].

In summary, we have constructed a simple model for emergence of fashions — goods that become popular not due to any intrisic value, but simply because "everybody wants it" —

in markets where people trade goods in order to fulfill the mutually frustrating demands of need and greed. This model shows spontaneous emergence of random products as money. The limitation of the model as an economic model is mainly due to the fact that it does not incorporate profit, but only works with perceptions of value. Therefore a detailed comparison with financial markets is not attempted. Nevertheless the model supports collectively driven fluctuations characterized by a Hurst exponent of ≈ 0.7, and its sets a frame where one may study how production/consumption influences development and collapse of value.

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Figure Captions

- Fig. 1. Agent trading activity as function of length of its memory in a market of 200 agents endowed with memories of sizes 1,2,...,200, or 5,10,...,1000. See text for more details.

- Fig. 2. Number of products that are not in the absorbing state as function of time, with time defined as number of trades per agent, for the cases of no production (full line), production rate $p = 0.001$ (short dashed line), and $p = 0.01$ (long dashed line). See text for the system parameters and other details.

- Fig. 3a. Time course of the demand $D(j)$ of a given product j, and of its volatility $V(j,t) \propto \cdot |D(j,t) - D(j,t-1)|/D(j,t-1)$. One observe that the demand has persistent fluctuations, and that the volatility $V(j)$ tends to cluster. The system parameters are the same of Fig. 2.

Fig. 3b. Hurst analysis of fluctuations in demand. In this case we considered a larger system, $N_{ag} = 500$, $N_{pr} = 500$, $N_{mem} = 1000$ and $N_{unit} = 1000$, thus spanning a larger scaling regime than for the smaller systems considered before. We see that the variation in demand may be fitted with a power law $\Delta D \propto (\Delta t)^H$, where H is the Hurst exponent, $H = 0.65 \pm 0.05$.

- Fig. 4. Product properties as function of how much they are in demand. The parameter values are the same of Fig. 2, with a production rate of $p = 0.001$. The upper curve shows the monetary value of the product, as defined in the text, which shows a transition to global acceptance for demand values above ≈ 600. The lower curves show the number of agents that do not posess the product, measured instantanously (middle curve), or over a time interval where each product is traded in average two times (lower curve).

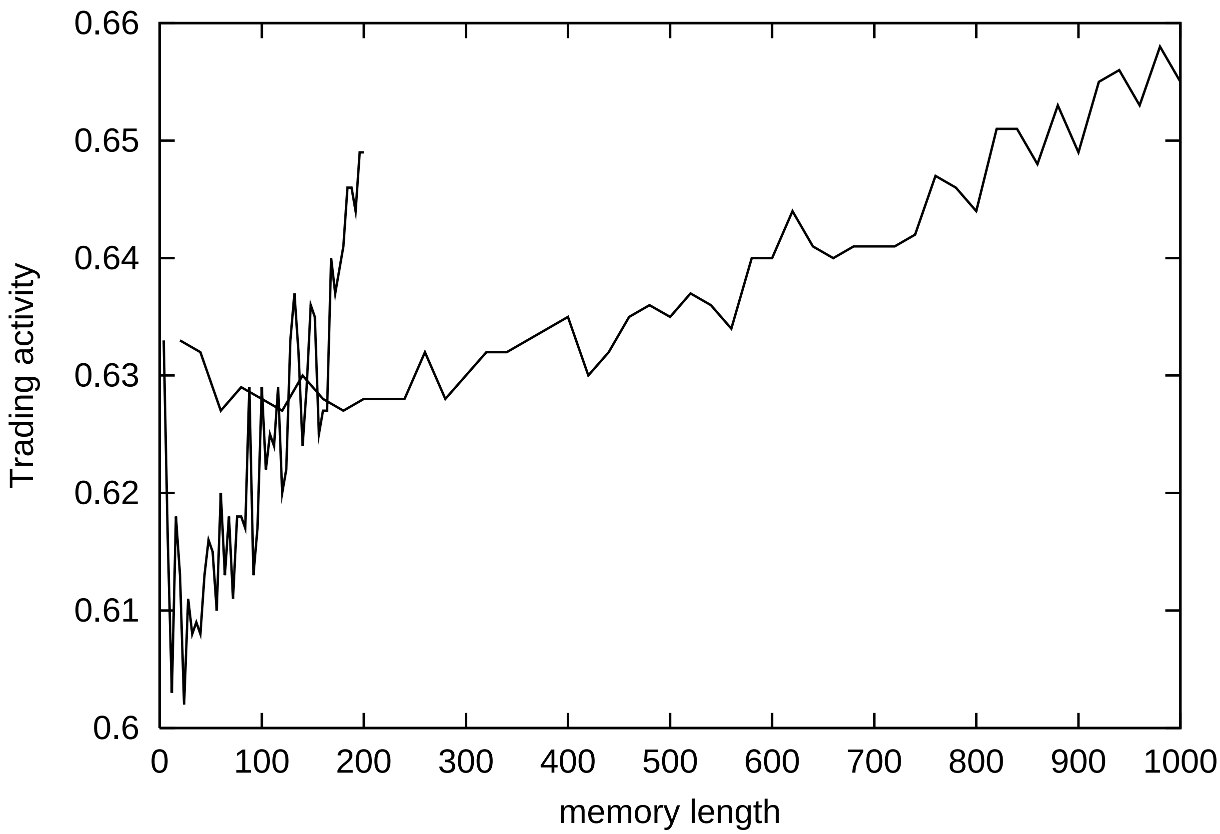

Figure 1

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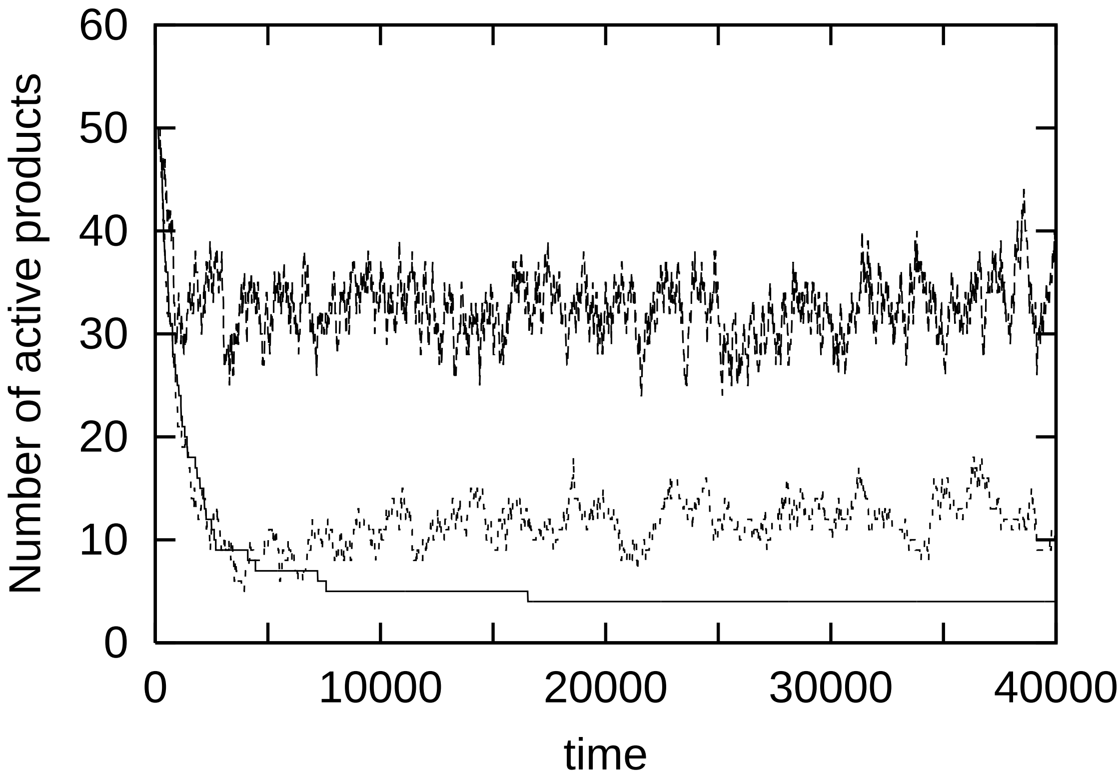

Figure 2

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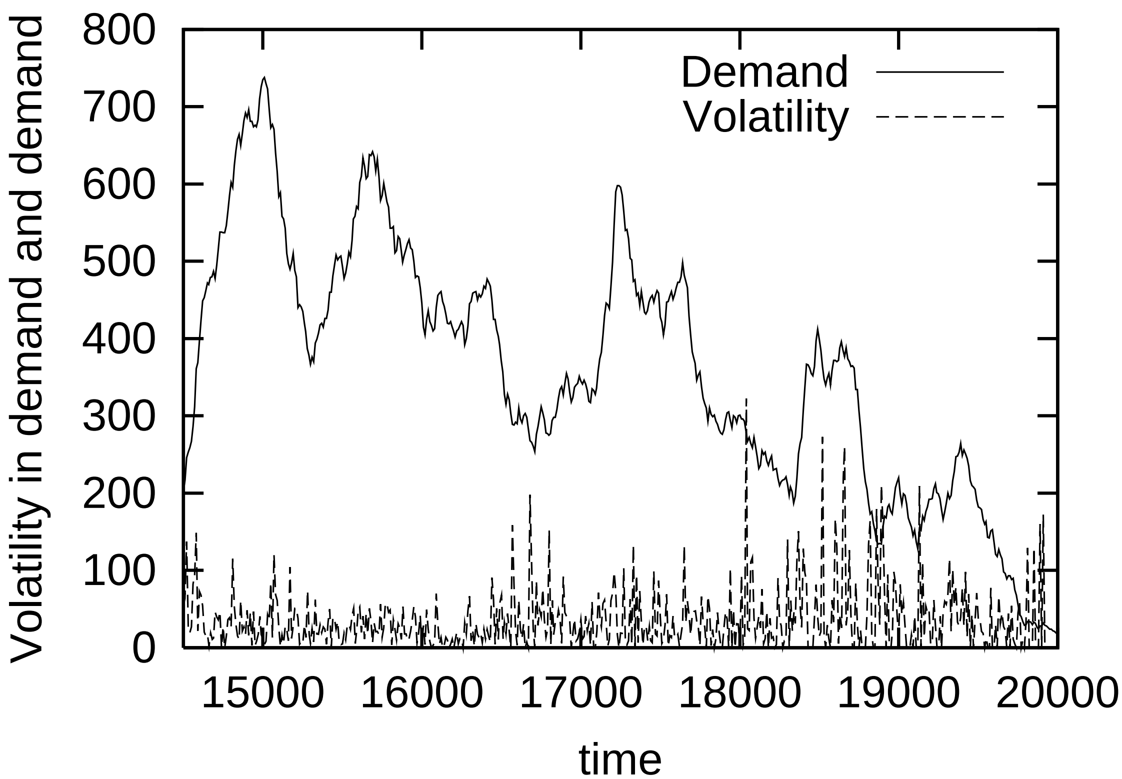

Figure 3a

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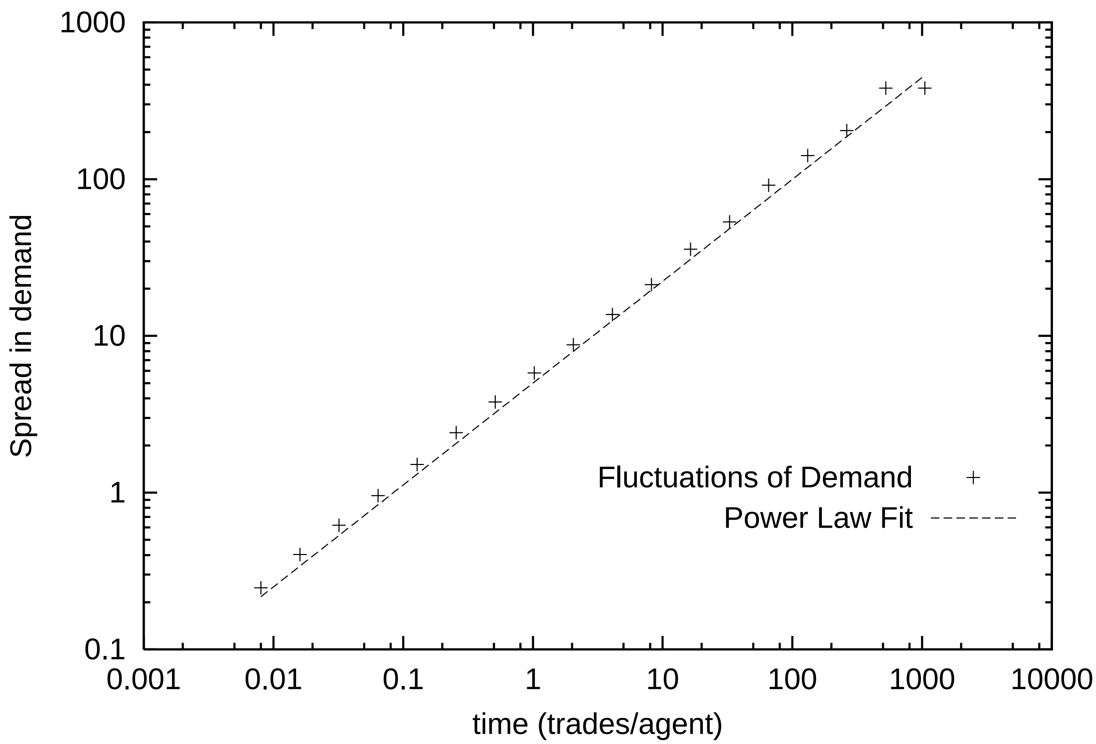

Figure 3b

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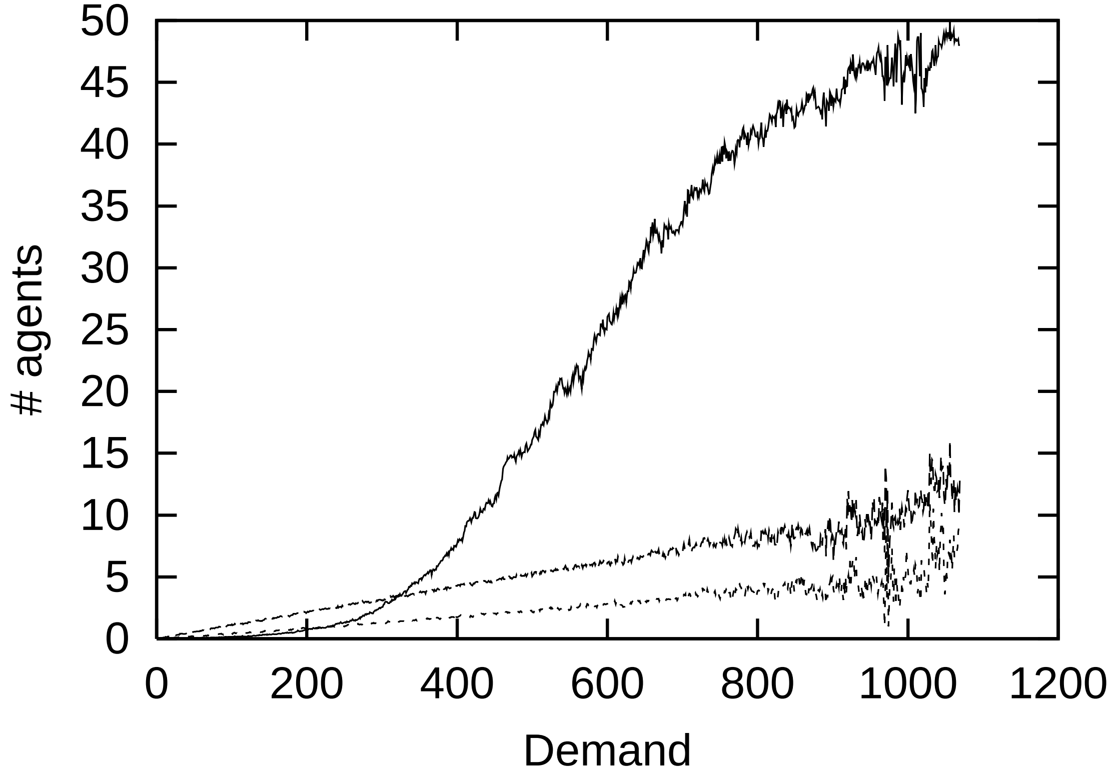

Figure 4

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